MATH 590: QUIZ 1

Name:

1. For $v_1, \ldots, v_r$ in the vector space V over F, define $\text{Span}\{v_1, \ldots, v_r\}$. (3 points)

Solution. $\text{Span}\{v_1, \ldots, v_r\}$ is the set of all linear combinations $\alpha_1 v_1 + \cdots + \alpha_r v_r$, with $\alpha_1, \ldots, \alpha_r \in F$.

2. Consider $(\mathbb{R}^+)^2$ with addition $(x_1, x_2) +' (y_1, y_2) := (x_1y_1, x_2y_2)$ and scalar multiplication given by $c * (x_1, x_2) := (x_1^c, x_2^c)$, for $c \in \mathbb{R}$. Verify the the following vector space axioms: (i) V has an additive identity; (ii) Vector addition is commutative; (iii) A sum of scalars distributes across a vector. (3 points)

Solution. The additive identity is (1, 1):

$$(x_1, x_2) +' (1, 1) = (x_1 \cdot 1, x_2 \cdot 1) = (x_1, x_2) = (1, 1) +' (x_1, x_2).$$

Addition is commutative:

$$(x_1, x_2) +' (y_1, y_2) = (x_1y_1, x_2y_2) = (y_1x_1, y_2x_2) = (y_1, y_2) +' (x_1, x_2).$$

A sum of scalars distributes across a vector: For $a, b \in \mathbb{R}$,

$$(a + b) * (x_1, x_2) = (x_1^{a+b}, x_2^{a+b}) = (x_1^a x_1^b, x_2^a x_2^b) = (x_1^a, x_2^a) +' (x_1^b, x_2^b) = a * (x_1, x_2) +' b * (x_1, x_2).$$

3. Let V denote the vector space of 2×2 matrices over $\mathbb{R}$ and W denote the set of matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ in V satisfying $3a - 2d = 0$. Show that W is a subspace of V. (4 points)

Solution. Suppose $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$ belong to W, so that $3a - 2d = 0 = 3e - 2h$. Then $A + B = \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$, and $3(a + e) - 2(d + h) = (3a - 2d) + (3e - 2h) = 0 + 0 = 0$, so that $A + B \in W$. For $\lambda \in \mathbb{R}$, $\lambda A = \begin{pmatrix} \lambda a & \lambda b \\ \lambda c & \lambda d \end{pmatrix}$ and $3(\lambda a) - 2(\lambda d) = \lambda(3a - 2d) = \lambda \cdot 0 = 0$, so that $\lambda A \in W$. Therefore, W is a subspace of V.